Erratum to *Parameterized Analysis of Reconfigurable Broadcast Networks*

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The proof of Theorem 2 published in [1] contains a mistake, kindly pointed out by Nicolas Waldburger.

The exact error is the following: to bound the norm of set $\mathcal{M}$, a configuration $C \in \mathcal{M}$ is considered where $C(q) \leq v(q) + N', \forall q$ for some N'. A configuration C_N is then defined such that $C_N(q) = \min(C(q), v(q) + N)$ for all q with the N given by Theorem 1. The proof then states that $C_N \in [\![\theta]\!]_N$, which is only possible if $C_N(q) \geq v(q) + N$ for all q. This is wrong because $C(q)$ may be smaller than $v(q) + N$ on some q, entailing $C_N(q) < v(q) + N$ and $C_N \notin [\![\theta]\!]_N$.

The results of the original paper up until Theorem 2 remain valid. We are currently working on a solution. Until it is found, we put below the part of Theorem 2 that still holds, which states that counting sets are closed under reachability. We also give a reduction from cube-reachability to unbounded initial cube reachability. Since unbounded initial cube reachability is decidable in PSPACE [2], this shows that cube-reachability is in PSPACE, which was the result of Theorem 4 now proved in another way.

Theorem 1 (Counting sets are closed under reachability). *Let $\mathcal{C}$ be a cube. Then $post^*(\mathcal{C})$ is a counting set. The same holds for pre^* by using the given RBN with reversed transitions.*

Proof. We start by defining a counting set $\mathcal{M}$ of configurations, which we will then prove to be equal to $post^*(\mathcal{C})$. Given a symbolic configuration θ of $post^*(\Delta_\mathcal{C})$, we define the set $\min(\theta, \mathcal{C})$ to be the set of configurations $C \in [\![\theta]\!]$ such that C is minimal for the order $\preceq_\theta$ over the configurations of $post^*(\mathcal{C})$, i.e.

$$\min(\theta, \mathcal{C}) = \min_{\preceq_\theta} \{C \in [\![\theta]\!] \mid C \in post^*(\mathcal{C})\}$$

We can now define $\mathcal{M}$ to be the following set

$$\mathcal{M} = \bigcup_{\theta \in post^*(\Delta_\mathcal{C})} \bigcup_{C \in \min(\theta, \mathcal{C})} \mathcal{C}_C^\theta,$$

where $\mathcal{C}_C^\theta$ is the cube $\mathcal{C}_{(C,S)}$ for S such that $\theta = (v, S)$. Since $\mathcal{M}$ is a finite union of cubes, it is a counting set.

We show that $post^*(\mathcal{C}) \subseteq \mathcal{M}$. Let $C \in post^*(\mathcal{C})$. There exists $C_0 \in \mathcal{C}$ such that $C_0 \xrightarrow{*} C$, and there exists $\theta_0 \in \Delta_\mathcal{C}$ such that $C_0 \in [\![\theta_0]\!]$. Applying Lemma

1, we obtain the existence of $\theta \in post^*(\theta_0) \subseteq post^*(\Delta_C)$ such that $C \in [\![\theta]\!]$. Now, there exists a configuration $C' \in \min(\theta, C)$ such that $C' \preceq_\theta C$. By definition of $\mathcal{C}^\theta_{C'}$, C is in $\mathcal{C}^\theta_{C'}$ and thus in $\mathcal{M}$.

Now we show that $\mathcal{M} \subseteq post^*(\mathcal{C})$. Let $C \in \mathcal{M}$. By definition, there must be a symbolic configuration $\theta \in post^*(\Delta_C)$ and a configuration $C' \in post^*(\mathcal{C})$ such that $C' \preceq_\theta C$. By the Compatibility Lemma (Lemma 3), C is in $post^*(\mathcal{C})$ as well.

This result also holds for $pre^*(\mathcal{C})$. If $\mathcal{R} = (Q, \Sigma, R)$ is the given RBN, consider the "reverse" RBN $\mathcal{R}_r$, defined as $\mathcal{R} = (Q, \Sigma, R_r)$ where R_r has a transition $(q, \star a, q')$ for $\star \in \{!, ?\}$ iff R_r has a transition $(q', \star a, q)$. Notice that $\mathcal{R}_r$ is still an RBN and that $post^*(\mathcal{C})$ in $\mathcal{R}$ is equal to $pre^*(\mathcal{C})$ in $\mathcal{R}_r$. $\square$

This is enough to show the closure theorem that followed Theorem 2 in [1], using also that counting sets are closed under boolean operations and that counting sets are finite unions of cubes.

Corollary 1 (Closure). *Counting sets are closed under $post^*$, pre^* and boolean operations.*

Now we present the reduction from cube-reachability to unbounded initial cube-reachability which will allow us to conclude like Theorem 4 of our paper [1]:

Theorem 2. *Cube-reachability is* PSPACE-*complete for RBN.*

Recall: in [2], the authors define a sub-class of the cube-reachability problem, which we call the *unbounded initial cube-reachability* problem. More precisely, the sub-class considered in [2] is the following: We are given an RBN and two cubes $\mathcal{C} = (L, U)$ and $\mathcal{C}' = (L', U')$ with the special property that $L(q) = 0$ and $U(q) \in \{0, \infty\}$ for every state q. We then have to decide if $\mathcal{C}$ can reach $\mathcal{C}'$. This problem was shown to be PSPACE-complete ([2], Theorem 5.5).

Reduction Let $\mathcal{P} = (Q, \Sigma, \delta)$ be an RBN and $\mathcal{C}_0, \mathcal{C}_1$ two cubes; $(\mathcal{P}, \mathcal{C}_0, \mathcal{C}_1)$ is our instance of cube reachability. We assume all components of $\mathcal{C}_0$ are of the form $0 \leq q < \infty$ or $a_q \leq q < b_q$.

We show that we can do this without loss of generality. Let $NZ(\mathcal{C})$ denotes the set of states q such that $a \leq q < \infty$ with $a > 0$ in $\mathcal{C}$. We construct $(\mathcal{P}', \mathcal{C}'_0, \mathcal{C}'_1)$ such that $NZ(\mathcal{C}'_0)$ is empty and $(\mathcal{P}', \mathcal{C}'_0, \mathcal{C}'_1)$ is a positive instance of cube-reachability if and only if $(\mathcal{P}, \mathcal{C}_0, \mathcal{C}_1)$ is. Let $\mathcal{P}', \mathcal{C}'_0, \mathcal{C}'_1$ be defined as follows: $Q' := Q \cup \{\bar{q} : q \in NZ(\mathcal{C}_0)\}$, and $\delta' := \delta \cup \{(\bar{q}, !e, q) : q \in Q\}$ for some letter $e \notin \Sigma$. Let $\mathcal{C}'_0 := \bigwedge_{q \in Q \setminus NZ(\mathcal{C}_0)} \mathcal{C}_0(q) \wedge \bigwedge_{q \in NZ(\mathcal{C}_0)} (\bar{q} = a_q \wedge 0 \leq q < \infty)$, and $\mathcal{C}'_1 := \mathcal{C}_1 \wedge \bigwedge_{q \in NZ(\mathcal{C}_0)} \bar{q} = 0$.

Lemma 1. $\exists \gamma_0 \in \mathcal{C}_0, \gamma_1 \in \mathcal{C}_1$ *such that* $\gamma_0 \xrightarrow{*}_\mathcal{P} \gamma_1$ *iff* $\exists \gamma'_0 \in \mathcal{C}'_0, \gamma'_1 \in \mathcal{C}'_1$ *such that* $\gamma'_0 \xrightarrow{*}_{\mathcal{P}'} \gamma'_1$.

Proof. ⇒) Take $t \stackrel{\text{def}}{=} (\bar{q}, !e, q)$ until $\bar{q}$ is empty, then follow the transitions of $\gamma_0 \stackrel{*}{\to}_{\mathcal{P}} \gamma_1$ in $\mathcal{P}'$. ⇐) By definition of $\mathcal{C}'_0$ and $\mathcal{C}'_1$, $\gamma'_0(\bar{q}) = a_q$ and $\gamma'_1(\bar{q}) = 0$. Since t is the only transition to and from $\bar{q}$, we can assume $\gamma'_0 \stackrel{*}{\to}_{\mathcal{P}'} \gamma'_1$ is of the form $\gamma'_0 \stackrel{t}{\to}_{\mathcal{P}'} \ldots \stackrel{t}{\to}_{\mathcal{P}'} \stackrel{\sigma}{\to}_{\mathcal{P}'} \gamma'_1$ where t is taken a_q times and σ is some sequence of transitions not containing t. Then we set $\gamma_0(q) = \gamma'_0(q) + \gamma'_0(\bar{q})$ if $q \in NZ(\mathcal{C}_0)$ and $\gamma_0(q) = \gamma'_0(q)$ otherwise. Configuration γ_1 is obtained by taking σ in $\mathcal{P}$ from γ_0. □

Hence, we can assume that all bounds of $\mathcal{C}_0$ are of the form $0 \leq q < \infty$, in which case q is called unbounded, or $a_q \leq q < b_q$ for some $b_q \in \natural$, in which case q is called bounded. The bounds of $\mathcal{C}_1$ are denoted $x_q \leq q < y_q$, without specifying whether y_q is ∞ or an integer. We note $U(\mathcal{C}_0)$ the set of unbounded states and $B(\mathcal{C}_0)$ the set of bounded states. Let $q_1, \ldots, q_k, \ldots, q_n$ be a numbering of the states of $\mathcal{P}$ such that the bounded states appear first, i.e. $q_1, \ldots, q_k$ are bounded and $q_{k+1}, \ldots, q_n$ are unbounded. Intuitively, we are going to construct $\mathcal{P}'$ as $k+1$ copies of $\mathcal{P}$. The first k copies will contain the agents that start in the bounded states $q_1, \ldots, q_k$ respectively, and the last copy will contain the agents that start in the unbounded states. The agents can receive broadcasts from other copies, but they stay in the states of their own copy.

Construction We construct $(\mathcal{P}', \mathcal{C}'_0, \mathcal{C}'_1)$ as follows, where k is the number of bounded states of $\mathcal{C}_0$: $\mathcal{P}'$ consists of $k+1$ copies of $\mathcal{P}$ denoted $\mathcal{P}_j$ for $j \in \{1, \ldots, k+1\}$. To denote the jth copy of a state q_i, we will write q_i^j and Q_j as the set $\{q^j : q \in Q\}$. We define $U_j(\mathcal{C}_0)$ (resp. $B_j(\mathcal{C}_0)$) as the set of copies in Q_j of the unbounded (resp. bounded) states in $\mathcal{C}_0$. We define transitions of $\mathcal{P}'$ as follows: $\delta' := \bigcup_{1 \leq i \leq k+1} \delta_i$ where for all $1 \leq i \leq k+1$, $\delta_i := \{(q^i, \_, p^i) : (q, \_, p) \in \delta\}$.

Let $\mathcal{C}'_1$ be the cube defined by

$$\bigwedge_{1 \leq j \leq k} [(0 \leq q_j^j < \infty) \wedge \bigwedge_{q \in Q_j \setminus \{q_j^j\}} (q = 0)] \wedge \bigwedge_{q \in U_{k+1}(\mathcal{C}_0)} (0 \leq q < \infty) \wedge \bigwedge_{q \in B_{k+1}(\mathcal{C}_0)} (q = 0)$$

For each $a_i \leq q_i < b_i$ with $i \in \{1, \ldots, k\}$, we guess m_i s.t. $a_i \leq m_i < b_i$. For each $i \in \{1, \ldots, k\}$ we guess a multiset $M_i := \{p_i^1, \ldots, p_i^{m_i}\}$ such that (1) for all $p \in M_i$, q_i can reach p in $\mathcal{P}$, meaning there is a sequence of transitions of δ of the form $q_i \to p_1 \to p_2 \ldots \to p$ where the labels do not matter; and (2) for all $q \in Q$, $\sum_{1 \leq i \leq k} M_i(q) < y_q$. For all $q \in Q$, let us note $l_q := \max(x_q - \sum_{1 \leq i \leq k} M_i, 0)$. Let $\mathcal{C}'_0$ be the cube defined by

$$\bigwedge_{1 \leq j \leq k} \bigwedge_{q \in Q_j} (q = M_j(q)) \wedge \bigwedge_{q \in U_{k+1}(\mathcal{C}_1)} (l_q \leq q < \infty) \wedge \bigwedge_{q \in B_{k+1}(\mathcal{C}_1)} (l_q \leq q < y_q - \sum_{1 \leq i \leq k} M_i(q))$$

Note that $y_q - \sum_{1 \leq i \leq k} M_i(q)$ may be ∞. The $(\mathcal{P}', \mathcal{C}'_0, \mathcal{C}'_1)$ thus constructed is an instance of unbounded initial cube-reachability.

Correctness Now we want to show that $(\mathcal{P}, \mathcal{C}_0, \mathcal{C}_1)$ is a positive instance of cube-reachability if and only if there exists $(m_i)_{1 \leq i \leq k}$ and $(M_i)_{1 \leq i \leq k}$ as above such that $(\mathcal{P}', \mathcal{C}'_0, \mathcal{C}'_1)$ is a positive instance of unbounded initial cube-reachability.

$\Rightarrow$) Suppose $(\mathcal{P}, \mathcal{C}_0, \mathcal{C}_1)$ is a positive instance of cube-reachability. Then $\exists \gamma_0 \in [\![\mathcal{C}_0]\!], \gamma_1 \in [\![\mathcal{C}_1]\!]$ such that $\gamma_0 \xrightarrow{*}_{\mathcal{P}} \gamma_1$. For all $q_i \in B(\mathcal{C}_0)$, set $m_i := \gamma_0(q_i)$. Let γ_0' be such that for all $q_i \in U(\mathcal{C}_0)$, $\gamma_0'(q_i^i) = m_i$, for all $q \in B(\mathcal{C}_0)$, $\gamma_0'(q^{k+1}) = \gamma_0(q)$, γ_0' is zero for every other state. Notice that $\gamma_0' \in \mathcal{C}_0'$, for $\mathcal{C}_0'$ defined as above. To define γ_1', we start by proving the following lemma:

Lemma 2. *Given γ_0, γ_1 and γ_0', if there exists a run $\gamma_0 \xrightarrow{*}_{\mathcal{P}} \gamma_1$, there exists γ_1' such that $\gamma_0' \xrightarrow{*}_{\mathcal{P}'} \gamma_1'$ and for all $q \in Q$, $\sum_{1 \leq i \leq k+1} \gamma_1'(q^i) = \gamma_1(q)$.*

Proof. We proceed by induction on the length of the run in $\mathcal{P}$. First note $c_0 \rightarrow_{\mathcal{P}} c_1 \rightarrow_{\mathcal{P}} \cdots \rightarrow_{\mathcal{P}} c_m$ with $c_0 = \gamma_0$ and $c_m = \gamma_1$. Let us show by induction on m that there exists a run $c_0' \rightarrow_{\mathcal{P}'} \cdots \rightarrow_{\mathcal{P}'} c_m'$ with $c_0' = \gamma_0'$, and at any point n, for all $q \in Q$, $\sum_{1 \leq i \leq k+1} c_n'(q^i) = c_n(q)$. For $n = 0$ it is immediate. For $n > 0$, let us assume that there is a run $c_0' \rightarrow_{\mathcal{P}'} \cdots \rightarrow_{\mathcal{P}'} c_n'$ such that $c_n \rightarrow_{\mathcal{P}'} c_{n+1}$ and $\sum_{1 \leq i \leq k+1} c_n'(q^i) = c_n(q)$ for all $q \in Q$. Let $(q_i, !a, q_j)$ be the broadcast transition from c_n to c_{n+1}. From c_n, we chose one $q_i^l \in c_n$ and change its state to q_j^l (i.e we apply one of the transitions $(q_i^l, !a, q_j^l)$). We are sure that there exists one as $\sum_{1 \leq i \leq k+1} c_n'(q^i) = c_n(q)$. This conserve the property because in each sides of the equality we remove one for q_i and add one for q_j. Now, we proceed the same way for each $(q, ?a, q')$ applied between c_n and c_{n+1}. From the construction of $\mathcal{P}'$ all these transitions appear also in each copy of $\mathcal{P}$ and therefore we can construct c_{n+1}' such that $c_n' \rightarrow_{\mathcal{P}'} c_{n+1}'$ and $\sum_{1 \leq i \leq k+1} c_{n+1}'(q^i) = c_{n+1}(q)$ for all $q \in Q$, which concludes the proof. □

Hence, we can pick such a γ_1', such that $\gamma_0' \xrightarrow{*}_{\mathcal{P}'} \gamma_1'$ and $\sum_{1 \leq i \leq k+1} \gamma_1'(q^i) = \gamma_1(q)$ for all $q \in Q$. Note that agents cannot move from one copy of $\mathcal{P}$ to another. Therefore, for each $i \in \{1, \ldots, k\}$, as there are m_i agents in copy i in γ_0', there are also m_i agents in copy i in γ_1'. We define M_i the multiset corresponding to configuration γ_1' restricted to the copy i. This allows to define $\mathcal{C}_1'$ as above. Let us show that $\gamma_1' \in \mathcal{C}_1'$. Since for every $q \in B(\mathcal{C}_1)$, $\sum_{1 \leq i \leq k+1} \gamma_1'(q^i) = \gamma_1(q)$, we have that:

$$x_q \leq \sum_{1 \leq i \leq k+1} \gamma_1'(q^i) < y_q$$

By subtracting $\sum_{1 \leq i \leq k} M_i(q)$ we get:

$$l_q \leq \gamma_1'(q^{k+1}) < y_q - \sum_{1 \leq i \leq k} M_i(q)$$

We can do the same reasoning for $q \in U(\mathcal{C}_1)$ by considering only the lower bound. Thus $\gamma_1' \in \mathcal{C}_1'$, which concludes the proof of the left to right implication.

$\Leftarrow$) Suppose there exists $(m_i)_{1 \leq i \leq k}$ and $(M_i)_{1 \leq i \leq k}$ such that $(\mathcal{P}', \mathcal{C}_0', \mathcal{C}_1')$ is a positive instance of unbounded initial cube reachability. We define γ_0 as follows: for all $q_i \in B(\mathcal{C}_0)$, $\gamma_0(q_i) = \gamma_0'(q_i^i)$ and for all $q \in U(\mathcal{C}_0)$, $\gamma_0(q) = \gamma_0'(q^{k+1})$. We now prove the following lemma:

Lemma 3. *Given γ_0', γ_1' and γ_0, if there exists a run $\gamma_0' \xrightarrow{*}_{\mathcal{P}'} \gamma_1'$, then there exists γ_1 such that $\gamma_0 \xrightarrow{*}_{\mathcal{P}} \gamma_1$ and for all $q \in Q$, $\sum_{1 \le i \le k+1} \gamma_1'(q^i) = \gamma_1(q)$.*

Proof. The proof is also an induction on the length of the run in $\mathcal{P}'$. As previously, note $c_0' \to_{\mathcal{P}'} \cdots \to_{\mathcal{P}'} c_m'$ with $c_0' = \gamma_0'$ and $c_m' = \gamma_1'$. We show by induction that there exists a run $c_0 \to_{\mathcal{P}} \cdots \to_{\mathcal{P}} c_m$ with $c_0 = \gamma_0$ and at any point n, for all $q \in Q$, $\sum_{1 \le i \le k+1} c_n(q^i) = c_n(q)$. For $n = 0$, it is immediate as $\gamma_0' \in \mathcal{C}_0'$. For $n > 0$, let us assume that there is a run $c_0 \to_{\mathcal{P}} \cdots \to_{\mathcal{P}} c_n$ with the desired property. Let $(q_i^j, !a, q_l^j)$ be the broadcast from c_n' to c_{n+1}'. From c_n, we apply the transition $(q_i, !a, q_j)$. We are sure that we can apply this transition as $\sum_{1 \le j \le k+1} c_n'(q_i^j) = c_n(q_i)$. Now for all the receptions that occur between c_n' and c_{n+1}', we do the same by applying the corresponding transitions in $\mathcal{P}$. Note that this preserves the equality. We constructed a c_{n+1} such that $c_n \to_{\mathcal{P}} c_{n+1}$ with the desired equality, which concludes the proof. $\square$

Hence, we can pick such a γ_1, such that $\gamma_0 \xrightarrow{*}_{\mathcal{P}} \gamma_1$ and $\sum_{1 \le i \le k+1} \gamma_1'(q^i) = \gamma_1(q)$ for all $q \in Q$. It follows that for all $q \in Q$, $x_q \le \gamma_1(q) < y_q$, and therefore $\gamma_1 \in \mathcal{C}_1$.

It remains to show that $\gamma_0 \in \mathcal{C}_0$. To this end, let us notice this: for an infinite constraint on a state q_j^i in $\mathcal{C}_0'$ (with $1 \le i \le k$), if there is no state q^i with an infinite constraint in $\mathcal{C}_1'$, then we can bound the initial constraint on q_j^i. Indeed, as one state q_j^i can only reach states q^i, if all the final cube is bounded on all of these reachable states, we can bound the constraint on q_j^i. One natural bound is $\sum_{q \in Q} y_{q^i}'$ where y_{q^i}' is the upper bound on the component of q^i in $\mathcal{C}_1'$. Notice now that for all $1 \le i \le k$, there is no unbounded components for all $q \in Q_i$ in $\mathcal{C}_1'$. Hence, for each q_i^i, even if $q_i^i \in U(\mathcal{C}_0')$, it holds that $\gamma_0'(q_i^i) < \sum_{q \in Q} y_{q^i}'$. In fact, as there is no new agents during the run and as $\forall q_i^j, j \ne i, \gamma_0'(q_i^j) = 0$, we have that $\gamma_0'(q_i^i) = \sum_{q \in Q} y_{q^i}' = |M_i|$ (recall that copies are disjoints). Hence, for all $1 \le i \le k+1$:

$$a_{q_i} \le \gamma_0'(q_i^i) < b_{q_i}$$

It is immediate that, for all $q \in B(\mathcal{C}_0)$,

$$a_q \le \gamma_0(q) < b_q$$

Hence, $\gamma_0 \in \mathcal{C}_0$, which conclude the proof.

PSPACE *membership.* This reduction leads to a **PSPACE** algorithm for deciding cube-reachability. Let $\mathcal{P}, \mathcal{C}_0, \mathcal{C}_1$, we want to decide the if $\mathcal{C}_0$ can reach $\mathcal{C}_1$. We guess $(m_i)_{1 \le i \le k}$ and $(M_i)_{1 \le i \le k}$ and construct $(\mathcal{P}', \mathcal{C}_0', \mathcal{C}_1')$ as above. We use the **PSPACE** algorithm from [2] to decide whether $\mathcal{C}_0'$ can reach $\mathcal{C}_1'$. This non-deterministic algorithm is in **PSPACE** because by Savitch's Theorem **PSPACE** = **NPSPACE**. In [2], the authors show that the unbounded initial cube-reachability is **PSPACE**-complete in the size of the input when the input is written in unary. In fact, one can encode the symbolic configurations (v, S) (first defined in [2]) by considering v as a vector of $|Q|$ components which are each bounded by $|Q| \times \|\mathcal{C}\|$,

where $\mathcal{C}$ is the input cube of the unbounded initial cube-reachability problem. As a consequence, with the right encoding of symbolic configurations, the algorithm presented in [2] works in polynomial space when the input is written in binary. Thus we conclude the same for the cube-reachability problem.

References

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